Foundations of Query Languages

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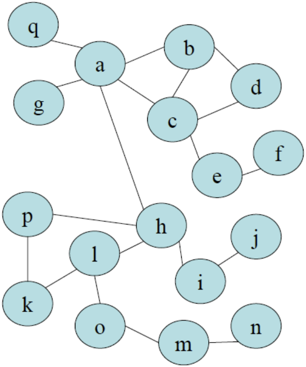
SS 2011

Intractable Problems

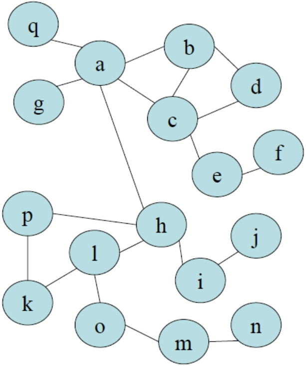


Definition

Independent Set (IS): Let $G = (V, E)$ be a graph. $V' \subseteq V$ is an independent set if and only if $\forall v_1, v_2 \in V'$, $(v_1, v_2) \notin E$, and $|V'|$ is maximal.

- The problem of Independent Set is NP-Complete.
- IS problem can be solved in linear time, if the underlying graph has bounded treewidth.

Tree decomposition

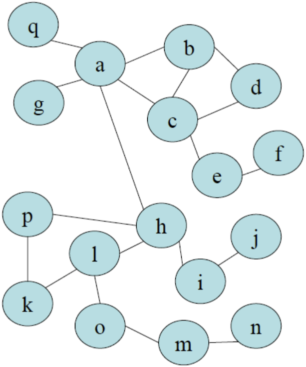


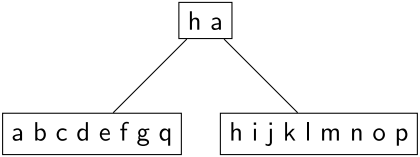
Tree Decomposition

Given a graph $G = (V, E)$, a tree decomposition of G is a tree T, in where every node X_i in T consists of a set of vertices in V, such that:

1. For every $v \in V$, there exists a tree node X_i in T, such that $v \in X_i$.
2. For every edge $(v, w) \in E$, there exists a tree node X_i containing both v and w.
3. For every v: the tree nodes that contain v form a connected subtree of T. (connectedness condition).

Tree decomposition





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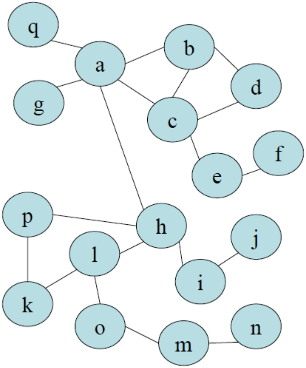

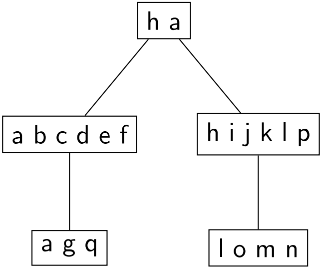

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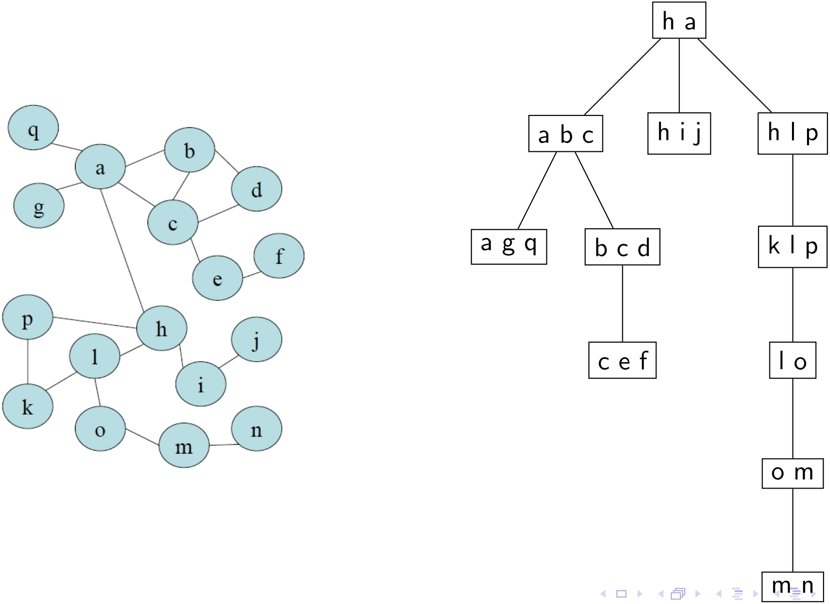

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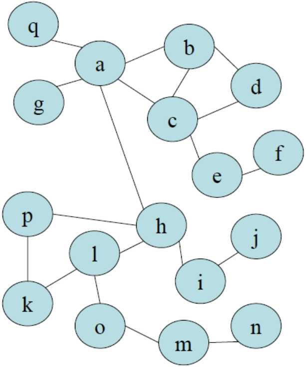
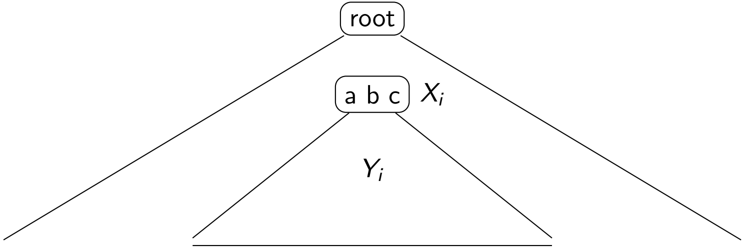


Treewidth

- The width of a tree decomposition T is $max(\{|X| - 1 : X \text{ node in } T\})$. (max node size -1)
- The treewidth is the minimum width over all tree decompositions of G.

IS Computation over tree decomposition



Given a node X_i in T, Y_i is the set of nodes that are descendants of X_i. $G(Y_i)$ be the graph induced by Y_i.

When we have an independent set W of $G(Y_i)$, and want to extend it to an independent set of G, then important is only what vertices in X_i belong to W, not what vertices in $Y_i - X_i$ belong to W. Of the latter, only the number of vertices in W is important.

Intuition: if a vertex $d \in Y_i$ and d occurs out of Y_i, then d must occur in X_i (connectedness condition!) As a consequence, those vertices in $Y_i - X_i$ will not affect the extended independent set any more.

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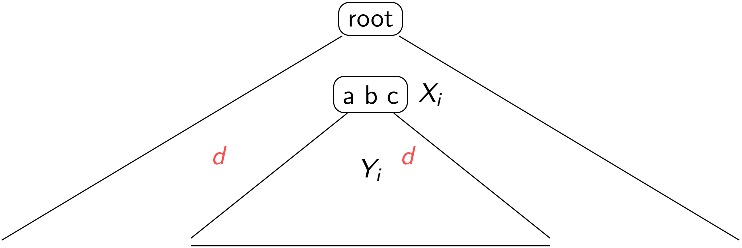

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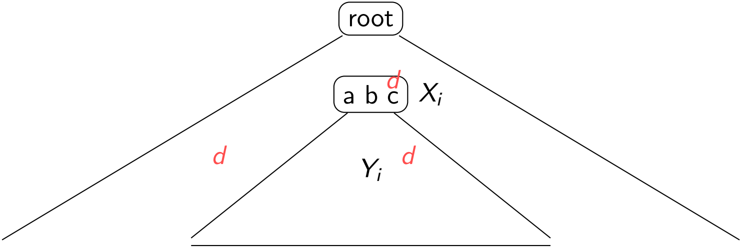

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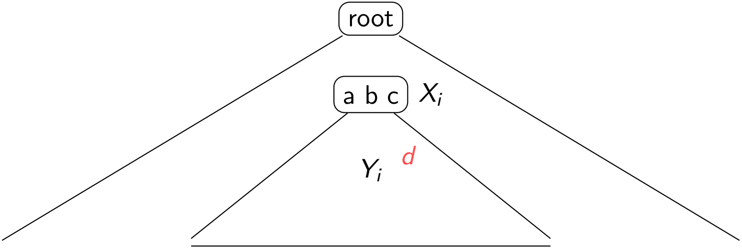

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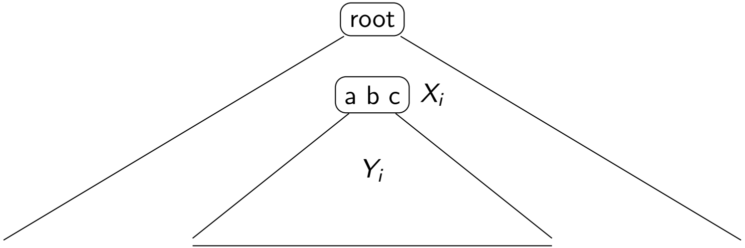
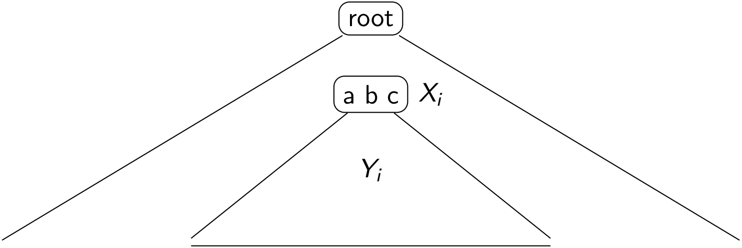


For $Z \subseteq X_i$, define $is_i(Z)$ to be the maximum size of the independent set in $G(Y_i)$ with $W \cap X_i = Z$.

IS Computation over tree decomposition

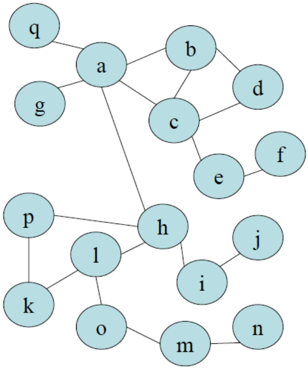


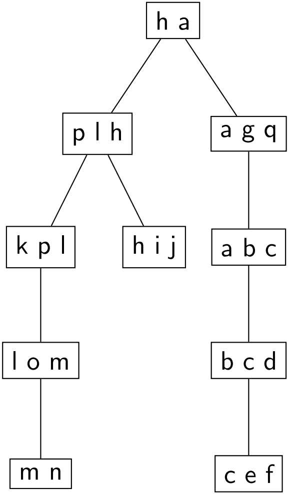
For a leaf node X_i, find all the maximum independent sets $Z \subseteq X_i$ from $G(X_i)$ and set $is_i(Z) = |Z|$.
For an internal node X_i with two children X_j and X_k, we set $is_i(Z) =$

$$max\{is_j(Z') + is_k(Z'') + |Z \cap (X_i - X_j - X_k)| - Z \cap X_j \cap X_k|\}$$

where $Z \cap X_j = Z' \cap X_i$, $Z \cap X_k = Z'' \cap X_i$ and $\forall v, w \in Z : (v, w) \notin E$.

Tree decomposition





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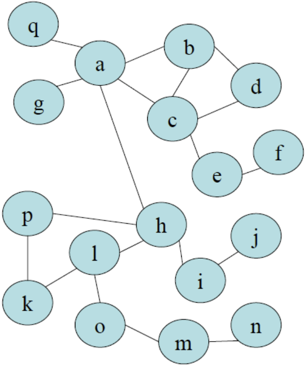

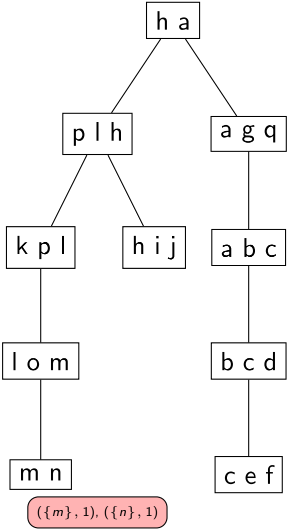

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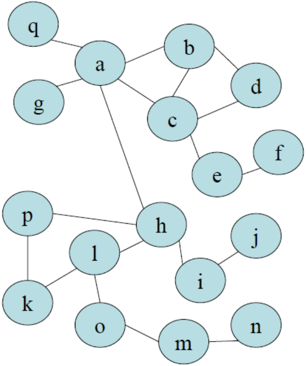

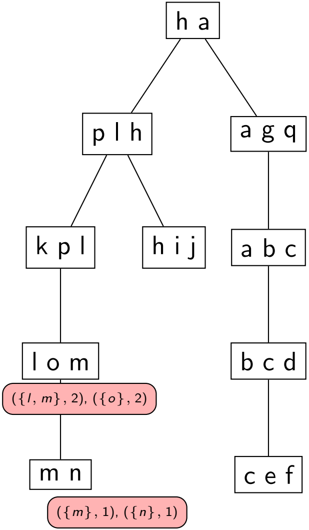

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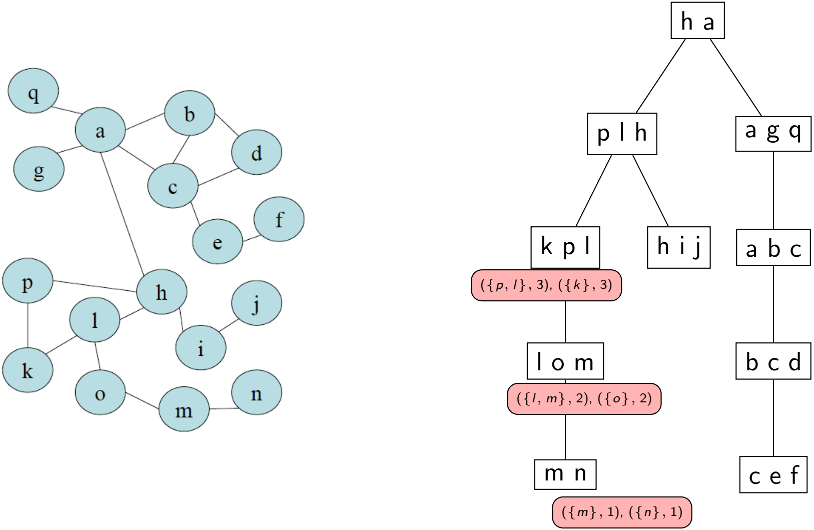

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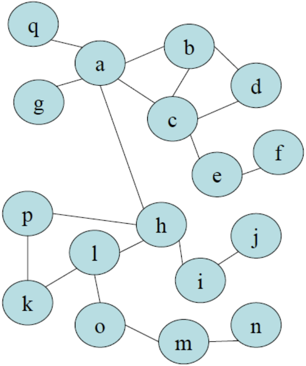

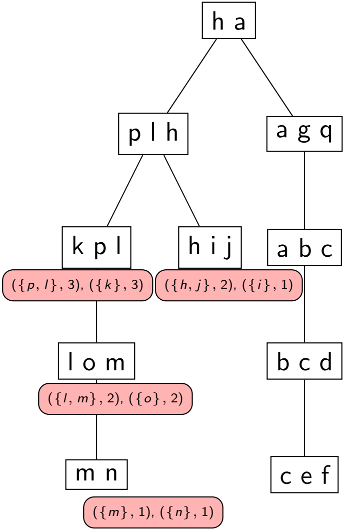

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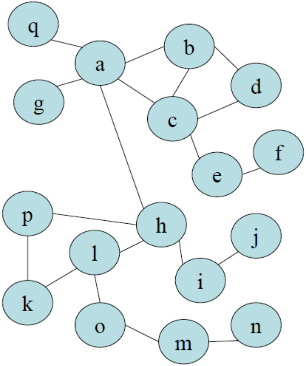

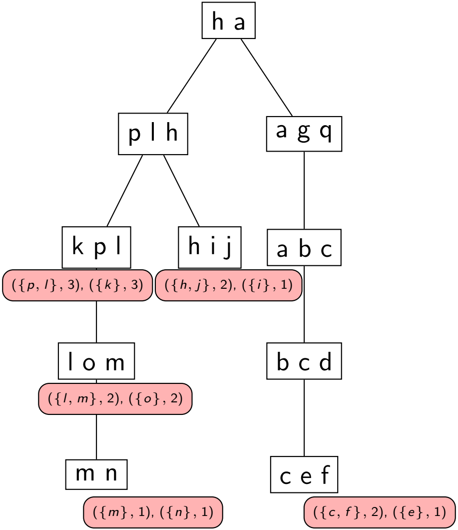

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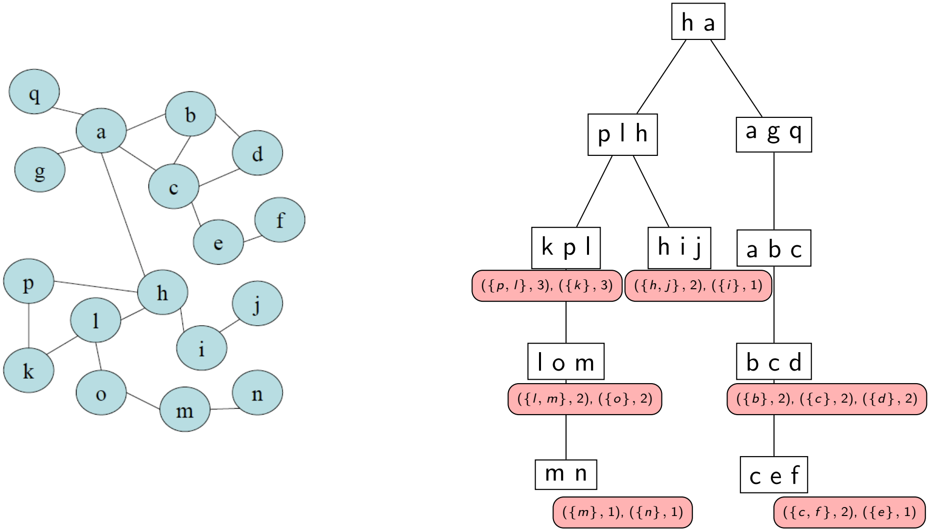

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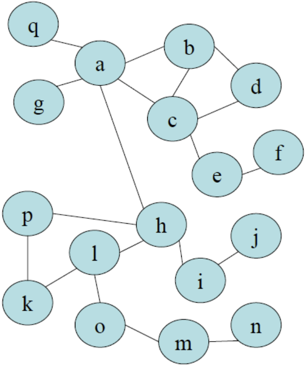

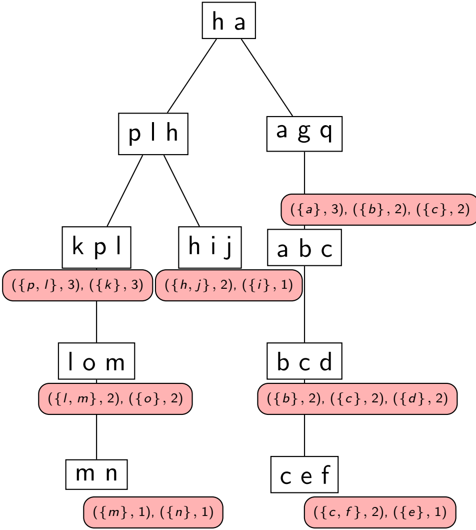

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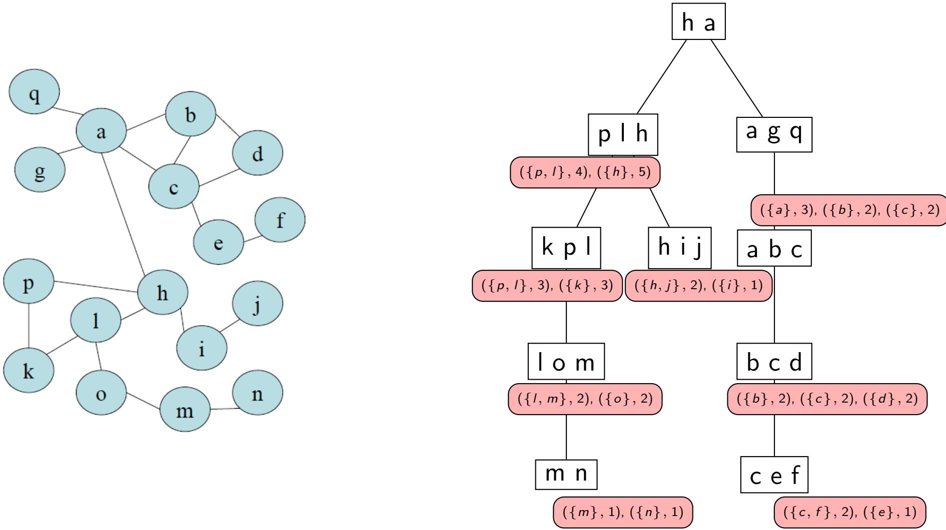

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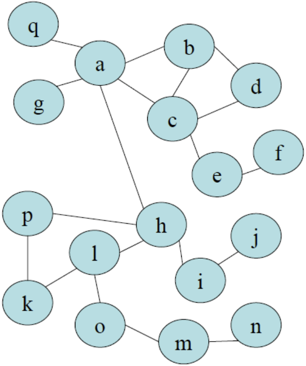

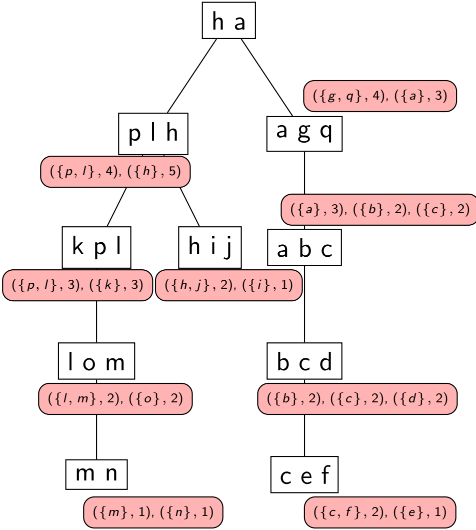

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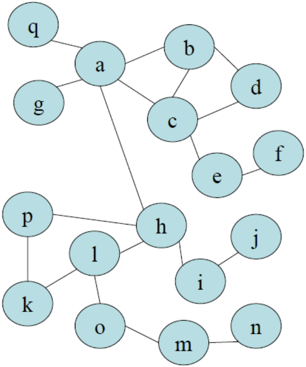
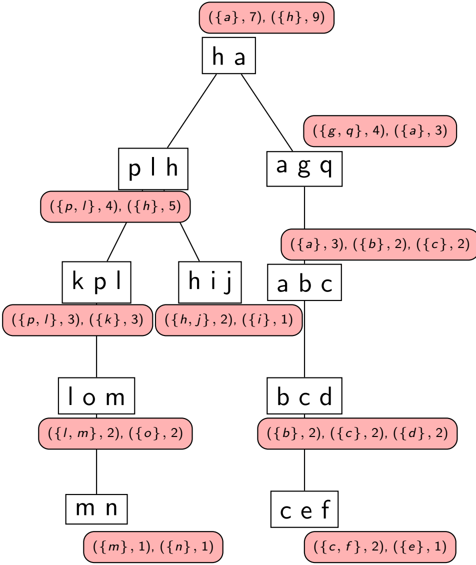
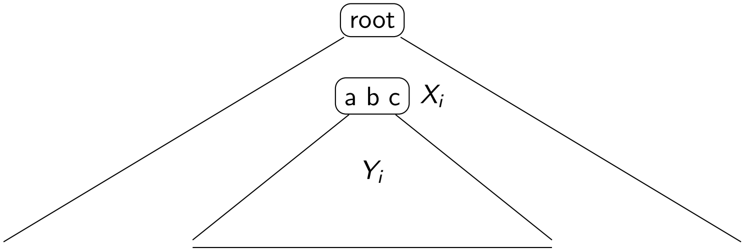




IS Computation over tree decomposition



So the overall time cost is $2^{tw} * |T|$ where tw is the treewidth of G.